

## Product of Linear Approximations

Suppose we have two complicated functions and we need an estimate of the value of their product. We could multiply the functions out and then approximate the result, or we could approximate each function separately and then find the product of the two (simple) approximations. Does it matter which we do? Not very much.

Prove that the linear approximation of  $f(x) \cdot g(x)$  equals the (linear part of the) product of the linear approximations of  $f(x)$  and  $g(x)$ .

## Linear approximation of a product

30/7/25

$$f(x)g(x) \approx f(x_0)g(x_0) + (f'(x_0)g(x_0) + f(x_0)g'(x_0))(x-x_0)$$

when  $x \approx x_0$

$$\Rightarrow f(x)g(x) \approx f(x_0)g(x_0) + f'(x_0)g(x_0)(x-x_0) + f(x_0)g'(x_0)(x-x_0)$$

## Product of two linear approximations

When  $x \approx x_0$ ,

$$f(x) \approx f(x_0) + f'(x_0)(x-x_0)$$

$$g(x) \approx g(x_0) + g'(x_0)(x-x_0)$$

$$\begin{aligned} \Rightarrow f(x)g(x) &\approx f(x_0)g(x_0) + (f'(x_0)(x-x_0))g(x_0) \\ &\quad + (g'(x_0)(x-x_0))f(x_0) + f'(x_0)g'(x_0)(x-x_0)^2 \\ &= f(x_0)g(x_0) + f'(x_0)g(x_0)(x-x_0) + f(x_0)g'(x_0)(x-x_0) \\ &\quad + f'(x_0)g'(x_0)(x-x_0)^2 \end{aligned}$$

$\therefore$  If  $(x-x_0)^2$  is negligible, the two approximations are similar